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M-MEMHS: Modified Minimization of Error in Multi Hop System for Localization of Unknown Sensor Nodes

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***Abstract*—Localization of sensor nodes is one of the important issues in Wireless Sensor Networks (WSNs). Location of node can be used as the location of occurrence of an event. Error handling and scalability are main research issues that need to be taken care of while estimating the efficiency of any localization algorithm. In this paper, we propose an approach of error correction mechanism on top of Minimization of Error in Multi Hop System (MEMHS) for Localization algorithm. MEMHS algorithm deals with a scalable error correction of multi-literate localization process using a few Geographical Positioning Systems (GPS) enabled nodes. The MEMHS authors assumed that error propagates linearly and is equal in any direction. In the present work the authors show that error propagates non-linearly with respect to hop count, and magnitude of error (X coordinate or Y coordinate) depends on the direction of equator lines. This paper proposes a modified algorithm of MEMHS, named as M-MEMHS. Furthermore, optimum deployment strategy is introduced so that maximum number of sensor nodes can be localized. By analyzing the proposed algorithm in comparison to MEMHS, it is found that the proposed algorithm has better performance in terms of error correction.**

***Index Terms*—Wireless Sensor Network, Localization, Triangulation, Multilateration, Error Correction, MEMHS.**

I. Introduction

A large number of applications demand the location of the sensing event. For example, an agriculture monitoring system demands for the location where the insects or pests are detected. Habitat monitoring of wild animals needs the location of animals. There are two approaches towards localization, like: Proximity based localization [1][2], and Range-based localization [3]. Proximity based localization assumes a graph model of the network. The network is represented by the graph $G(V,E)$, where V represents the vertex (i.e, nodes of the network) and E represents the edges (i.e, link between those nodes). It is assumed that a set of nodes H which is sub-set of nodes V is location aware. The cardinality of set H is assumed to be m and the cardinality of set V is assumed to be n. Therefore, the total number of location un-aware nodes is $n-m$. The goal is to find out the location of unknown nodes $(V-H)$ with respect to the location aware nodes H. Different range-based localization techniques use Received Signal Strength Indicator (RSSI), Time based method (ToA, TDoA) [4][5], Angle of Arrival (AoA) [5][6] etc. Localization algorithms like Range-free 3D node localization [7], and Stochastic Algorithms for 3D Node Localization [8] are shown to be effective in different application environments. In [9], the authors had provided an efficient Meta-heuristic Range Based Node Localization for WSNs. The above works provide in-depth findings and their appropriate applications. The authors in [10] discussed localization algorithm based on H-best Particle Swarm Optimization (HPSO). The work in [10] shows the tradeoff between accuracy and fast convergence. One of the most common methods of knowing location of any sensor node is to have Geographical Positioning System (**GPS)** with each sensor node. But that is not feasible because **GPS** devices are costly and consume high power. Moreover, the size of the GPS device is large. Hence, there is a need for finding methods that will reduce the number of GPS devices to be used in a particular situation. However, the accompanying challenge faced with lesser number of GPS devices, is obtaining accuracy of location of non-GPS nodes. The motivation, therefore, is to have an affordable solution to localization with the challenge of minimizing errors during the process.

In this paper, we propose further error correction mechanism over Minimization of Error in Multi Hop System (MEMHS) algorithm [11]. MEMHS is an RSSI based [8] localization technique. Error creeps in during calculation of location, thereby making the location inaccurate and it is thus one of the major problems in case of localization. This paper aims to minimize the error of MEMHS algorithm further by presenting an optimum node deployment strategy for optimizing the entire scenario.

The organization of the rest of the paper is as follows. Section II surveys major techniques of localization. Section III

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describes MEMHS algorithm. The limitation of MEMHS is discussed in Section IV. The present work on M-MEMHS algorithm for minimizing the localization error is discussed in Section V. Section VI validates the M-MEMHS algorithm. Section VII discusses optimum node deployment strategy for maximizing the number of nodes to localize and minimize the error. Section VIII discusses simulation results and compares M-MEMHS algorithm with MEMHS algorithm and multilateration algorithm [9]. Section IX provides a brief comparison of the proposed scheme with previous related schemes. Section X concludes the paper along with future directions.

II. RELATED WORK

In [10], the algorithms for localization are divided in two categories: centralized and distributed. There are different algorithms to localize a sensor node using centralized approach. The examples include Multi-Dimensional Scaling (MDS) map [11]. In case of Distributed localization the sensor nodes compute their locations using their own resources like memory, processor. The Distributed localization algorithm can be categorized as: Beacon-based distributed algorithms, Relaxation-based distributed algorithms, coordinate system stitching based distributed algorithm, Hybrid localization algorithms, Interferometric ranging based localization, Error propagation aware localization. In Diffusion based distributed algorithm, individual node calculates the Centroid position of its location aware neighbor nodes. Approximate point-in-triangulation test (APIT) [12] is an example of Diffusion based localization algorithm. In case of Bounding Box localization approach, a rectangular region is formed by the nodes as its range of location. The Collaborative Multilateration localization is described in [13][14], which is an example of the Bounding Box [15] localization technique. A Gradient based localization algorithm is described in [16]. Among the sensor nodes, some sensor nodes are GPS-enabled sensor nodes called 'Seeds'. Initially, each seed node establishes a gradient by sending a message with its location information and the hop count is set to one. After receiving the message along with the location information and hop count, the neighbor nodes re-broadcast the message to their neighbor nodes. The hop count represents the minimum hop distance from the 'Seed' node. Hybrid localization algorithm is a combination of more than one localization techniques. Hybrid localization is aimed to reduce the complexity. The localization scheme by Multi Dimensional Scaling (MDS) and Proximity Based Map (PBM) and Simple Hybrid Absolute-Relative positioning (SHARP) are examples of Hybrid localization algorithm. Multilateration [7] is the common process to know the location of non-GPS enabled node. Due to erroneous assumption of path loss coefficient and permittivity constant, the estimated distance may be erroneous. Therefore, the estimated location information will be erroneous. The error will propagate hop by hop with cumulative effect. MEMHS algorithms reduced the cumulative property of the error. This paper presents further error correction technique and provides a deployment strategy for optimum result. Table 1 provides the description of different symbols used in the paper. The error correction and deployment strategy will be discussed on top of MEMHS algorithm.

III. MEMHS ALGORITHM

MEMHS is an error correction algorithm for multilateration algorithm. Initially, all the nodes will calculate their location information. Thereafter, MEMHS algorithm finds out the more accurate location with respect to multilateration algorithm, which is described in [7]. The statement of MEMHS algorithm is 'If the approximated hop distance is multiplied from a set of beacon nodes A with the estimated localized value with respect to a set of beacon nodes B, and added to the product of the approximated hop distance from set B with the estimated localized value with respect to set A and the entire sum divided by the sum of the approximated hop distance from set A of nodes and set B of nodes, then the approximated error free localized value of any particular unknown node can be found'. The theoretical background of MEMHS algorithm is briefly discussed in the next Section.

TABLE I
DESCRIPTION OF DIFFERENT SYMBOLS

Symbol	Quantity
p_k	A point in the network area
p_k^{Ae}	Point p_k with respect to A set of beacon nodes
Φ_i^{Ae}	Generalized representation of x or y coordinates with respect to A set of beacon nodes
$\Delta\Phi_i^{Ae}$	Erroneous part of x or y coordinate with respect to A set of beacon nodes
$\Delta\Phi_{avg}^{Ae}$	Generalized representation of average of errors of coordinates x or y.
α, β, θ	Different angles
$\Delta\alpha, \Delta\beta, \Delta\theta$	Very small angle
h_A, h_B	Average hop count from A and B sets of beacon nodes
$\Phi(h)$	X or Y coordinate of a point at average hop distance h with respect to centroid of GPS enabled node.
$E_{\Phi[h]}$	Cumulated error at the coordinate point $\Phi(h)$

A. Error minimization mechanisms for MEMHS algorithm

Let us assume that the error due to the error factor at the i^{th} hop and j^{th} hop be denoted as $\Delta\Phi_i^{Ae}$ and $\Delta\Phi_j^{Ae}$ respectively where $\Delta\Phi_i^{Ae} / \Delta\Phi_j^{Ae} > 0$ (since the sign of error is same for both cases). Fig. 1 describes the scenario. From [7] we get the expression for the $p_k^{Ae}(x_k^{Ae}, y_k^{Ae})$:

$$\Phi_k^{Ae} = \Phi_k + (-1)^n \sum_{i=1}^{h_A} \left|\Delta\Phi_i^{Ae}\right| \text{; here } \Phi = \{x, y\} \qquad (1)$$

Let the average value of $\{\Delta\Phi_i^A\}_{i=1}^{h_A}$ be $\Delta\Phi_{avg}^A$. Therefore, the expression of p_k^{Ae} will be

$$\Phi_k^{Ae} = \Phi_k + (-1)^n h_A \left|\Delta\Phi_{avg}^{Ae}\right| \quad (2)$$

As per (2) the equation for p_k^{Be} will be as follows (here, the sign of error will be opposite with respect to *A* set of nodes):

$$\Phi_k^{Be} = \Phi_k + (-1)^{(n+1)} h_B \left|\Delta\Phi_{avg}^{Be}\right| \quad (3)$$

If the sensor nodes can be deployed uniformly over the region and errors are opposite with respect to neutral point [7], then we can say:

$$\frac{\sum_{i=1}^{h_A}\left|\Delta\Phi_i^{Ae}\right|}{h_A} \approx \frac{\sum_{i=1}^{h_B}\left|\Delta\Phi_i^{Be}\right|}{h_B}$$
$$\text{or,} \left|\Delta\Phi_{avg}^{Ae}\right| \approx \left|\Delta\Phi_{avg}^{Be}\right| \quad (4)$$

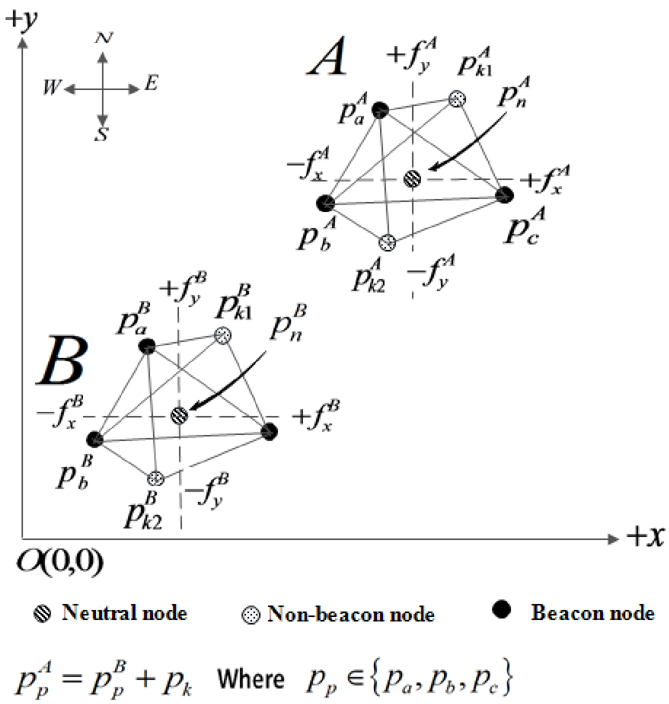

Fig. 1. Reflexive node position of two sets of nodes *A* and *B* [7].

If the approximated error free localized information is denoted as $p_k^{MEMHS}(x_k^{MEMHS}, y_k^{MEMHS})$, then the expression for the $p_k^{MEMHS}(x_k^{MEMHS}, y_k^{MEMHS})$ will be:

$$\Phi_k^{MEMHS} = \frac{\Phi_k^{Ae} h_B + \Phi_k^{Be} h_A}{(h_A + h_B)}$$
or,
$$\Phi_k^{MEMHS} = \Phi_k + (-1)^n \frac{h_B h_A (\left|\Delta\Phi_{avg}^{Ae}\right| - \left|\Delta\Phi_{avg}^{Be}\right|)}{h_A + h_B} \quad (5)$$

In [7] it has been proved that the estimated value of localized information of p_k is p_k^{MEMHS} and it is more accurate than the approximated values p_k^{Ae} and p_k^{Be}.

IV. Limitation of MEMHS Algorithm

MEMHS algorithm assumes that one hop error at any point is equal to others irrespective of angle between the non-beacon node with respect to centroid of referential beacon node and equator lines. But, in reality, the magnitude of error also depends on the angle of deployment. Section IV.A shows the relation between magnitude of error and angular position of a node. Moreover, it is assumed that, error in case of MEMHS will propagate with respect to the hop count in the additive cumulative way. However, error does not propagate in linear fashion, rather error propagates in nonlinear or exponential way, as observed. Section IV.B deals with non-linear nature of error with respect to hop count.

A. Magnitude of error with respect to angular positions

In Fig. 2 we have p_a, p_b and p_c to be the GPS enabled nodes, p_r and p_s to be the non-GPS nodes, p_u to be the neutral point. Also we assume that p_r^{fx}, p_s^{fx} and p_r^{fy}, p_s^{fy} are orthographic projections of the points p_r and p_s respectively on the axes f_x and f_y. The angles between $p_u p_s$ and f_x axis and angle between $p_u p_r$ and f_x axis are β and α respectively.

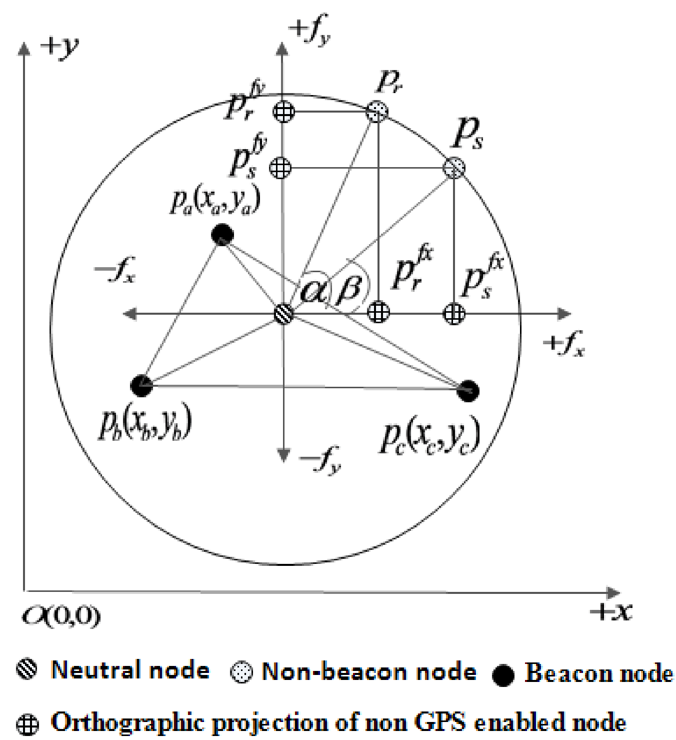

Fig.2. The change in magnitude of errors with changing angular position of node with respect to the neutral point keeping the radial distance same.

As per Fig. 2, it can be said that if radial distance is kept constant from the neutral point then we can write $f_{\Phi}^{\eta} = f_{\max}\sin(q\pi/2-\theta)$ where $\eta=\{r,s\}$, $\Phi=\{x,y\}$, $\theta=\{\alpha,\beta\}$ and $f_{\max}$, f_{Φ}^{η} are described in [7]. Here the value of $f_{\max}$ is the value of the circle. Moreover, if $\Phi = x$, then $q=2m+1$, else if $\Phi = y$ then, $q=2m$ where m is any finite and real number. As per Fig. 2, $\alpha>\beta$, where $\alpha,\beta<90^o$ then $\sin\alpha>\sin\beta$ and $\cos\alpha<\cos\beta$. Thereafter, $f_x^s > f_x^r$ and $f_y^s < f_y^r$. Let the total value of error in case of the point p_p be $\Delta\Phi_p^e$ where ξ is error factor. The description of f_{Φ}^{p} and s_{Φ}^{p} is given in Section IV.B. Then the expression for $\Delta\Phi_p^e$ is:

$$\Delta\Phi_p^e = \frac{f_{\Phi}^{p}(1-\xi^2)}{s_{\Phi}^{p}}$$ Where p_p is any arbitrary point

$$\Delta\Phi_p^e = \frac{f_{\max}^{p}(1-\xi^2)\left|\cos\theta_p\right|}{s_x^p} \tag{6}$$

Here θ_p is the angle between $p_p p_u$ and f_x axis

$$\Delta x_p^e = \Delta x_{\max}^e\left|\cos\theta_p\right|$$

Here $\Delta x_{\max}^e$ is the maximum error factor (keeping radial distance or distance from neutral point to point p_p constant). Let us assume that the estimated erroneous **X** co-ordinate of any point p_k is x_k^{Ae}. Thus, the expression for x_k^{Ae} will be:

$$x_k^{Ae} = x_k + (-1)^n \sum_{i=1}^{h_A}\left|\Delta x_{\max}^e\right|\left|\cos\theta_i^{Ae}\right| \tag{7}$$

$$x_k^{Ae} = x_k + (-1)^n h_A\left|\Delta x_{\max}^e\right|\left|\cos\theta_{avg}^{Ae}\right| \tag{8}$$

Similarly,

$$y_k^{Ae} = y_k + (-1)^n h_A\left|\Delta y_{\max}^e\right|\left|\sin\theta_{avg}^{Ae}\right| \tag{9}$$

By combining (8) and (9) we get:

$$\Phi_k^{Ae} = \Phi_k + (-1)^n h_A\left|\Delta\Phi_{\max}^e\right|\left|\sin(\frac{q\pi}{2}-\theta_{avg}^{Ae})\right| \tag{10}$$

If $\Phi_k^{Ae} = x_k^{Ae}$ then, $q=2m+1$ else, if $\Phi_k^{Ae} = y_k^{Ae}$ then, $q=2m$.

$$\Phi_k^{Be} = \Phi_k + (-1)^{n+1} h_A\left|\Delta\Phi_{\max}^{Be}\right|\left|\sin(\frac{q\pi}{2}-\theta_{avg}^{Be})\right| \tag{11}$$

If $\Phi_k^{Be} = x_k^{Be}$ then, $q=2m+1$ else, if $\Phi_k^{Be} = y_k^{Be}$ then, $q=2m$. Therefore, from equation (11) we can say that the magnitude of error also depends on the angular position of nodes and that has not been considered in the MEMHS algorithm. Fig. 3 shows the erroneous angular position of a node p_k estimated with respect to two different sets of beacon nodes. Due to error factor, cumulative error will be generated and for that reason, the point p_k will get two different estimated positions with respect to two different sets of beacon nodes. With respect to A set of beacon nodes, the estimated position is denoted as p_k^A and the angle with the X axis is (180^0+θ_{pk}^A). Similarly, with respect to B set of beacon nodes, the estimated position is denoted as p_k^B and the angle with the X axis is θ_{pk}^B. Average hop count from A and B sets of nodes to the point p_k is h_A and h_B respectively. Intuitively we can say that either erroneous angular position with respect to X axis will be greater than the actual angular position or it will be lesser than the actual angular position for both cases (with respect to A and B sets of nodes). Here, directed curved lines $p_n^A p_k^A$ and $p_n^B p_k^B$ indicate that angles of arrival after each hop are getting cumulative error. The lines ($p_n^A p_k^A$ and $p_n^B p_k^B$) are curved due to the cumulative error after each hop. Since the nodes are in the same direction with respect to GPS enabled nodes, the cumulative error is also either increasing or decreasing monotonically, so the curves are smooth.

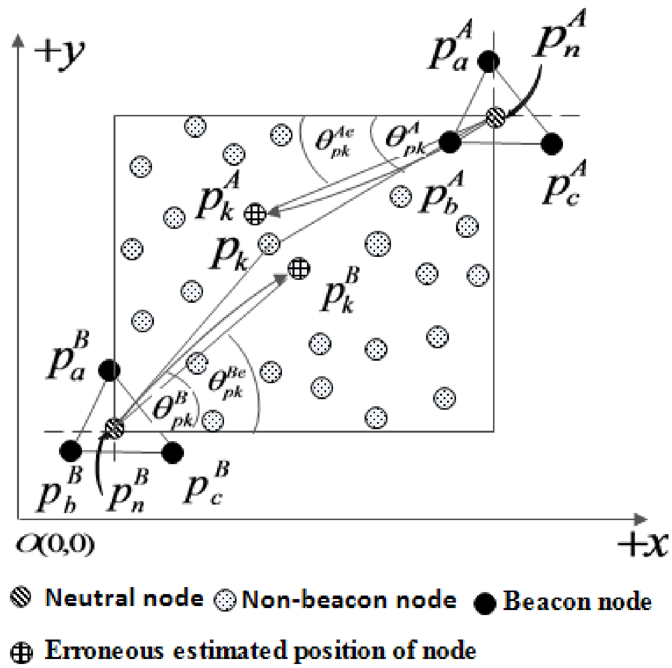

Fig.3. Angular error due to error factor

From (7) if we generalize the equation, then we can get

$$\Phi_k^{Ae} = \Phi_k + (-1)^n \sum_{i=1}^{h_A}\left|\Delta\Phi_{\max}^e\right|\left|\cos\theta_i^{Ae}\right| \tag{12}$$

with respect to A set of beacon nodes. The addition in the equation is vector addition. After averaging, we get resultant vector where the value of scalar part is

$h_A\left|\Delta\Phi^{e}_{\max}\right|$ and vector part is $\sin\left(\frac{q\pi}{2}-\theta^{Ae}_{avg}\right)$, which is the resultant vector after h_A number of hops. Whereas as per Fig.3, the erroneous position of node p_k with respect to the A set of beacon nodes is p^{A}_{k}. The angle between the point p^{A}_{k} and negative X axis is θ^{Ae}_{pk}. Conversely, we can say that θ^{Ae}_{pk} is the resultant angle after h_A number of hops. Therefore, from the above discussion, it can be said that, angle θ^{Ae}_{avg} and angle θ^{Ae}_{pk} are same. Similarly, we can say angle θ^{Be}_{avg} and angle θ^{Be}_{pk} are also same.

B. Exponential relationship of error with respect to hop count

In MEMHS algorithm, the error in the position of a node was assumed to be a linear function of hop count. But in reality, considering aspects of the triangulation method, error is found to be not linearly related to hop count. The generic formula of triangulation method discussed in [11][21].

$$x^{'}_{k}=\left(g^{k}_{x}+f^{k}_{x}\xi^{2}\right)/\,s^{k}_{x} \tag{13}$$

Where

$$g^{k}_{x}=\left\{\left(x^{2}_{a}+y^{2}_{a}\right)-\left(x^{2}_{c}+y^{2}_{c}\right)\right\}\left(y_{b}-y_{a}\right) -\left\{\left(x^{2}_{a}+y^{2}_{a}\right)-\left(x^{2}_{b}+y^{2}_{b}\right)\right\}\left(y_{c}-y_{a}\right) \tag{14}$$

$$f^{k}_{x}=\left(y_{a}-y_{c}\right)d^{2}_{b,k}+\left(y_{c}-y_{b}\right)d^{2}_{a,k} +\left(y_{b}-y_{a}\right)d^{2}_{c,k} \tag{15}$$

$$s^{k}_{x}=x_{b}\left(y_{c}-y_{a}\right)+x_{a}\left(y_{b}-y_{c}\right)+x_{c}\left(y_{a}-y_{b}\right) \tag{16}$$

$$\text{And } y^{'}_{k}=\left(g^{k}_{y}+f^{k}_{y}\xi^{2}\right)/\,s^{k}_{y} \tag{17}$$

$$g^{k}_{y}=\left\{\left(x^{2}_{a}+y^{2}_{a}\right)-\left(x^{2}_{c}+y^{2}_{c}\right)\right\}\left(x_{b}-x_{a}\right) -\left\{\left(x^{2}_{a}+y^{2}_{a}\right)-\left(x^{2}_{b}+y^{2}_{b}\right)\right\}\left(x_{c}-x_{a}\right) \tag{18}$$

$$f^{k}_{y}=\left(x_{a}-x_{c}\right)d^{2}_{b,k}+\left(x_{c}-x_{b}\right)d^{2}_{a,k} +\left(x_{b}-x_{a}\right)d^{2}_{c,k} \tag{19}$$

$$s^{k}_{x}=y_{b}\left(x_{c}-x_{a}\right)+y_{a}\left(x_{b}-x_{c}\right)+y_{c}\left(x_{a}-x_{b}\right) \tag{20}$$

As evident from equation (13), and equation (17), the generic equation of triangulation function is related to g function, f function and s function, and, therefore, the current node error will depend only on the measured distance and error caused due to erroneous measurement of distance. We can represent equation (13) and equation (17) in a generalized way as:

$$\Phi^{'}_{k}=\left(g^{k}_{\Phi}+f^{k}_{\Phi}\xi^{2}\right)/\,s^{k}_{\Phi}\text{ where }\Phi=\{x,y\} \tag{21}$$

Initially, we can compute the g, f and s functions from GPS enabled nodes and hop counts (if number of GPS enabled node is zero). We can represent g, f and s functions at zero hop count as $g_{\Phi[0]}$, $f_{\Phi[0]}$ and $s_{\Phi[0]}$. We can determine $\Phi[1]$ (where $\Phi[1]$ is the computed X and Y coordinates while average hop distance from GPS enabled node is one) with respect to $g_{\Phi[0]}$, $f_{\Phi[0]}$ and $s_{\Phi[0]}$. The equation for $\Phi[1]$ is:

$$\Phi[1]=\left(g_{\Phi[0]}+f_{\Phi[0]}\xi^{2}\right)/\,s_{\Phi[0]}$$

Similarly, we can determine $\Phi[h+1]$ with respect to $g_{\Phi[h]}$, $f_{\Phi[h]}$ and $s_{\Phi[h]}$. The expression for $\Phi[h+1]$ will be:

$$\Phi[h+1]=\left(g_{\Phi[h]}+f_{\Phi[h]}\xi^{2}\right)/\,s_{\Phi[h]} \tag{22}$$

where $\Phi[h]$ and $\Phi[h+1]$ are coordinates of a point at hop counts h and $h+1$. In equation (22), $\Phi[h+1]$ can be derived by g, f, s functions and ξ. Also g, f and s functions can be derived from $\Phi[h]$. Therefore, if we can start with k value from 0, then we can get the expression for $\Phi[1]$ as follows:

$$\Phi[1]=\left(g_{\Phi[0]}+\xi^{2}f_{\Phi[0]}\right)/\,s_{\Phi[0]} \tag{23}$$

Initially, the functions $g_{\Phi[0]}$, $f_{\Phi[0]}$ will be free from error because those values will be derived by using the coordinates of beacon nodes. Due to the error factor(ξ) [7], the value of $\Phi[1]$ will be erroneous. We assume the error propagation function for $\Phi[h]$ can be represented by $E_{\Phi[h]}$. The error $E_{\Phi[h-1]}$ will start at the hop value one. The value of $E_{\Phi[h]}$ will be cumulative of the previous hop error function $E_{\Phi[h-1]}$ and current hop error. As this accumulation also propagates, the expression $E_{\Phi[h]}$ will form a polynomial on the value ξ. From equations (13) and (17), it can be said, that, current hop error will be proportional to the previous hop error and $\xi^{2}d^{2}$ in case of f function.

$$E_{\Phi[h]}\alpha E_{\Phi[h-1]}\xi^{2}d^{2}$$

From equation (13) and (17), it can be said, that, the present hop error for g function is proportional to the previous hop error

$$E_{\Phi[h]}\alpha E_{\Phi[h-1]}$$

If we combine two equations, then we get the final recursive equation for $E_{\Phi[h]}$.

$$E_{\Phi(h)}=k_{1}E_{\Phi(h-1)}+k_{2}d^{2}_{h}\xi^{2}E_{\Phi(h-1)} \tag{24}$$

where k_1, k_2 are coefficients, d_h is the average distance at hop count h and ξ is the error factor. When the value of h

is equal to 1, then the mod value of error ($E_{\Phi[1]}$) will be $\left|\xi^2-1\right| d^2 f_{\Phi[1]}$. The expression of $E_{\Phi[2]}$ is:

$$E_{\Phi[2]} = E_{\Phi[1]} + \xi^2 d^2 E_{\Phi[1]} \tag{25}$$

If we put the value of $E_{\Phi[1]}$, then we get the expression $E_{\Phi[2]}$ as:

$$E_{\Phi[2]} = \left(\xi^2-1\right) d^2 f_{\Phi[1]} \left(1+\xi^2 d^2\right)$$
$$E_{\Phi[2]} = f_{\Phi[1]} \left(\xi^2 d^2 - d^2 + \xi^4 d^4 - d^4 \xi^2\right) \tag{26}$$

If we assume that $\xi >> 1$ then $\left|d^4 \xi^4\right| >> \left|d^2\xi^2 - d^2 - d^4\xi^2\right|$ after approximation we can re write the equation (26) as

$$E_{\Phi[2]} = f_{\Phi[1]} d^4 \xi^4 \tag{27}$$

Therefore, if we expand equation (27), we shall get the polynomial equation of $d\xi$ as discussed in equation (28):

$$E_{\Phi(h)} = \sum_{i=1}^{h} s_i (d\xi)^{2i} \tag{28}$$

where s_i is the coefficient of the i^{th} term. The approximated representation of equation (28) is:

$$E_{\Phi(h)} = c_h \left(d\xi\right)^{2(h+1)} \tag{29}$$

where c_h is the approximated coefficient when the hop count is h, and for simplicity we can write:

$$E_{\Phi(h)} = (t_h h)^{h+1} \tag{30}$$

where $t_h h = c_h \left(d\xi\right)^2$. Here, t_h is the unknown factor

From equation (30), we can say that the hop count is exponentially related to the cumulated error. If we assume $\xi \approx 1$ or $\xi << 1$ then also the relation between $E_{\Phi[h]}$ and h will not change (as per equation (30), only the value of coefficient (t_h) will change.

V. M-MEMHS ALGORITHM FOR MINIMIZING THE LOCALIZATION ERROR

In the previous section, we have shown that hop count is exponentially related to the cumulated error (equation (30)). But, in MEMHS algorithm the hop counts are considered to have linear relationship with the cumulative error. Therefore, the modified MEMHS algorithm considers the exponential relation between the cumulative error and hop count. In this paper we propose the Modified MEMHS (M-MEMHS) algorithm. We propose Theorem 1 in this paper, which is called Modified MEMHS (M-MEMHS) algorithm.

Theorem1:

Dividing the expression

$\Phi_k^{Ae} h_B^{h_B+1} \sin\left(q\pi/2-\theta_{pk}^{Be}\right) + \Phi_k^{Be} h_A^{h_A+1} \sin\left(q\pi/2-\theta_{pk}^{Ae}\right)$

by the expression

$h_A^{h_A+1} \sin\left(q\pi/2-\theta_{pk}^{Ae}\right) + h_B^{h_B+1} \sin\left(q\pi/2-\theta_{pk}^{Be}\right)$ will effectively neutralize the effect of error (observed the nature of the error as exponential power of hop count) and hence the location will be more appropriate than MEMHS algorithm. Thus the expression for the coordinates (incorporating the neutralizing effect of error) in case of M-MEMHS algorithm is:

$$\Phi_k^M = \frac{\Phi_k^{Ae} h_B^{h_B+1} \sin(\frac{q\pi}{2} - \theta_{pk}^{Be}) + \Phi_k^{Be} h_A^{h_A+1} \sin(\frac{q\pi}{2} - \theta_{pk}^{Ae})}{h_A^{h_A+1} \sin\left(\frac{q\pi}{2} - \theta_{pk}^{Ae}\right) + h_B^{h_B+1} \sin\left(\frac{q\pi}{2} - \theta_{pk}^{Be}\right)} \tag{31}$$

where $\Phi_k^M = \{x_k^M, y_k^M\}$, h_A and h_B are hop distances from *A* and *B* sets of beacon nodes, θ_{pk}^{Ae} and θ_{pk}^{Be} are the erroneous angular distances from *A* and *B* sets of beacon nodes, $q = \{m, 2m\}$, where m is any natural number.

Below is described the M-MEMHS algorithm.

A. M-MEMHS Algorithm

1. Consider two different sets of GPS enabled nodes (each set contains three nodes), referred to as set *A* and set *B*, which are placed at two opposite boundaries of WSN.
   Multilateration is applied and
   a. Compute co-ordinates $\left(\Phi_k^{Ae}\right)$ with respect to set A beacon nodes and the value $\left(h_A^{h_A}\right)$ are computed.
   b. Compute the approximated angle θ_{pk}^{Ae} with respect to A set of beacon nodes.
2. Multilateration is applied and
   a. Compute co-ordinates $\left(\Phi_k^{Be}\right)$ with respect to set B beacon nodes and the values $\left(h_B^{h_B}\right)$ are computed.
   b. Compute the approximated angle θ_{pk}^{Be} with respect to B set of beacon nodes.
3. Modified error free coordinates after applying the M-MEMHS algorithm are given by:

$$\Phi_k^M = \frac{\Phi_k^{Ae} \times h_B^{h_B+1} \sin(\frac{q\pi}{2} - \theta_{pk}^{Be}) + \Phi_k^{Be} h_A^{h_A+1} \sin(\frac{q\pi}{2} - \theta_{pk}^{Ae})}{h_A^{h_A+1} \sin\left(\frac{q\pi}{2} - \theta_{pk}^{Ae}\right) + h_B^{h_B+1} \sin\left(\frac{q\pi}{2} - \theta_{pk}^{Be}\right)}$$

where $\Phi_k^M = \{x_k^M, y_k^M\}$ and $q = \{1,2\}$,

VI. VALIDATION OF THEOREM 1

Let us assume, that, the error in case of MEMHS and M-MEMHS algorithm be denoted by E_{MEMHS} and $E_{M-MEMHS}$

respectively. If we consider approximated nonlinear error function to the MEMHS and M-MEMHS functions, then we can get the expressions of E_{MEMHS} and $E_{M-MEMHS}$ in equation (32) and (33) respectively.

$$E_{MEMHS}=\frac{\left|\begin{array}{l} h_B h_A^{h_A+1}\left(t_h^A\right)_m^{h_A+1}\sin\left(\frac{q\pi}{2}-\theta_{avg}^{Ae}\right) \\ -h_A h_B^{h_B+1}\left(t_h^B\right)_m^{h_B+1}\sin\left(\frac{q\pi}{2}-\theta_{avg}^{Be}\right)\end{array}\right|}{\left|h_A+h_B\right|} \quad (32)$$

Here the value of t_h^A depends on the angle of deployment. In the equation, $\left(t_h^A\right)_m$ is the maximum possible value of t_h^A at hop distance h_A, where $q=\{1,2\}$. In case of X coordinate, the value of q is 1 and in case of Y coordinate, the value of q is equal to 2.

$$E_{M-MEMHS}=\frac{\left|\begin{array}{l} h_B^{h_B+1}h_A^{h_A+1}\left(t_h^A\right)_m^{h_A+1}\sin\left(\frac{q\pi}{2}-\theta_{avg}^{Ae}\right)\sin\left(\frac{q\pi}{2}-\theta_{pk}^{Be}\right) \\ -h_A^{h_A+1}h_B^{h_B+1}\left(t_h^B\right)_m^{h_B+1}\sin\left(\frac{q\pi}{2}-\theta_{avg}^{Be}\right)\sin\left(\frac{q\pi}{2}-\theta_{pk}^{Ae}\right)\end{array}\right|}{\left|h_A^{h_A+1}\sin\left(\frac{q\pi}{2}-\theta_{pk}^{Ae}\right)+h_B^{h_B+1}\sin\left(\frac{q\pi}{2}-\theta_{pk}^{Be}\right)\right|} \quad (33)$$

Table II lists different conditions of measuring performance of MEMHS and M-MEMHS algorithms. We need to validate that M-MEMHS algorithm is more efficient than MEMHS algorithm with respect to different conditions mentioned in Table II.

TABLE II
DIFFERENT CASES FOR ANALYZING MEMHS ANDM-MEMHSERROR FUNCTION

Cond	h_A VS h_B	Relation between θ_{pk}^{Ae} and θ_{pk}^{Be}
1.	$h_A >> h_B$	When $h_A >> h_B$ then $\max\left|\theta_{pk}^{Ae}-\theta_{pk}^{Be}\right|\approx\frac{\pi}{4}$, $\theta_{pk}^{Ae}>\theta_{pk}^{Be}$ and $\sin\left(q\pi/2-\theta_{pk}^{Be}\right)\approx 0$
2.	$h_A >> h_B$	When $h_A >> h_B$ then $\max\left|\theta_{pk}^{Ae}-\theta_{pk}^{Be}\right|\approx\frac{\pi}{4}$, $\theta_{pk}^{Be}>\theta_{pk}^{Ae}$ and $\sin\left(q\pi/2-\theta_{pk}^{Ae}\right)\approx 0$
3.	$h_A \approx h_B$	$\theta_{pk}^{Ae}\approx\theta_{pk}^{Ae}$
4.	$h_B >> h_A$	When $h_B >> h_A$ then $\max\left|\theta_{pk}^{Ae}-\theta_{pk}^{Be}\right|\approx\pi/4$ $\theta_{pk}^{Ae}>\theta_{pk}^{Be}$ and $\theta_{pk}^{Be}=0$
5.	$h_B >> h_A$	When $h_B >> h_A$ then $\max\left|\theta_{pk}^{Ae}-\theta_{pk}^{Be}\right|\approx\pi/4$, $\theta_{pk}^{Be}>\theta_{pk}^{Ae}$ and $\theta_{pk}^{Ae}=0$

A. Condition 1

When $h_A >> h_B$, $\max\left|\theta_{pk}^{Ae}-\theta_{pk}^{Be}\right|\approx\pi/4$, $\theta_{pk}^{Ae}>\theta_{pk}^{Be}$ and $\theta_{pk}^{Be}=0$, then we can ignore h_B with respect to h_A. Now, we can say, that, the value of (or expression) E_{MEMHS} will be:

$$E_{MEMHS}=\left|h_B h_A^{h_A+1}\left(t_h^A\right)_m\sin\left(q\pi/2-\theta_{avg}^{Ae}\right)\right| \quad (34)$$

Here, we can assume, $\theta_{avg}^{Ae}\approx\theta_{pk}^{Ae}$ and $\theta_{avg}^{Be}\approx\theta_{pk}^{Be}$, therefore, we can go for approximations $\sin\left(q\pi/2-\theta_{avg}^{Ae}\right)\approx\sin\left(q\pi/2-\theta_{pk}^{Ae}\right)$ and $\sin\left(q\pi/2-\theta_{avg}^{Be}\right)\square\sin\left(q\pi/2-\theta_{pk}^{Be}\right)$. Thus, we can replace the angles θ_{avg}^{Ae} and θ_{avg}^{Be} with angles θ_{pk}^{Ae} and θ_{pk}^{Be} respectively. Since $\sin\left(q\pi/2-\theta_{avg}^{Be}\right)=0$, we can get the expression for $E_{M-MEMHS}$ as:

$$E_{M-MEMHS}=0 \quad (35)$$

If we compare equation (29) and equation (30) then we can easily say that:

$$E_{M-MEMHS}<E_{MEMHS}$$

B. Condition 2

Considering case2, the value (or expression) of E_{MEMHS} algorithm will be:

$$E_{MEMHS}=\left|h_B^{h_B+1}t_A^{h_B+1}\sin\left(q\pi/2-\theta_{avg}^{Be}\right)\right| \quad (36)$$

Since $\sin\left(q\pi/2-\theta_{pk}^{Ae}\right)\approx 0$, here, we can say, $\theta_{avg}^{Ae}\approx\theta_{pk}^{Ae}$ and $\theta_{avg}^{Be}\approx\theta_{pk}^{Be}$ (As per discussion in IV.A) then the value (or expression) of $E_{M-MEMHS}$

$$E_{M-MEMHS}\approx 0 \quad (37)$$

Therefore, it is clear, that, M-MEMHS is more efficient than MEMHS algorithm in case of condition 2.

C. Condition 3

As per condition 3, the hop count h_A is equal to h_B and angle θ_{avg}^{Ae} is equal to θ_{avg}^{Be}. Considering condition 3, we can say, that, the value of E_{MEMHS} and $E_{M-MEMHS}$ will both be zero. The rest of the conditions in Table 2 (Condition 4, Condition 5) are similar to the conditions Condition 1 and Condition 2. Therefore, from the above discussion, we can state that, overall, M-MEMHS is much more efficient than MEMHS algorithm.

VII. THE DEPLOYMENT STRATEGY OF GPS ENABLED NODES IN CASE OF MEMHS ALGORITHM

Here in Fig. 4, there are two sets of GPS enabled nodes, set *A* and set *B*. As per previous discussion with respect to these two sets of nodes (set *A* and set *B*), we can localize the entire sets of non-GPS enabled wireless sensor network (WSN) nodes. As per M-MEMHS algorithm, the necessary condition is that all the nodes need to be placed on the opposite co-ordinate with respect to co-ordinates (f_x^A, f_y^A) and (f_x^B, f_y^B). Thus, any WSN node needs to be placed within the co-ordinate of $(-1)^r f_x^A$, $(-1)^s f_y^A$ and $(-1)^{r+1} f_x^B$, $(-1)^{s+1} f_y^A$, where $r, s \in \{1,2,3\}$. This strategy is followed here.

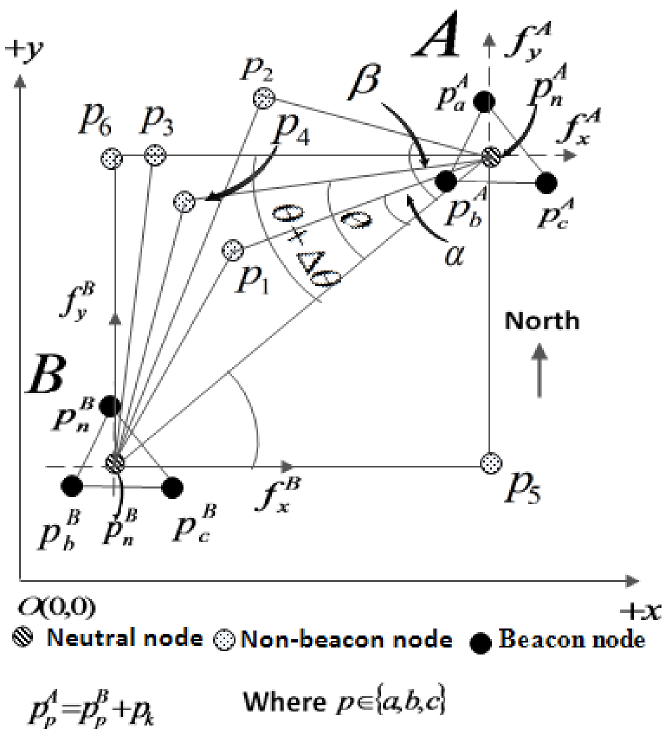

Fig. 4. The deployment strategy of GPS enabled nodes

In Fig. 4 we assume that, $\angle p_4 p_n^A p_n^B = \theta$, $\angle p_n^A p_n^B p_6 = \theta + \Delta\theta$, $\angle p_n^A p_n^B p_1 = \alpha$ and $\angle p_n^A p_n^B p_2 = \beta$ where $\alpha < \theta < \theta + \Delta\theta < \beta$. Also, as per Fig. 4, the points p_1 and p_4 are laid within the co-ordinates $(-1)^r f_x^A, (-1)^s f_y^A$ and $(-1)^{(r+1)} f_x^A, (-1)^{(s+1)} f_y^A$.

In Fig. 4, if $\Delta\theta \to 0$, then we can consider $p_n^A p_3$ and $p_n^A p_4$ to be on the same straight line. Also, we can say, that line $p_n^A p_3$ and f_x^A axis denote the same straight line. Then the computational error of **X** co-ordinate of the nodes, which is deployed along the f_x co-ordinate (with respect to the *A* set of GPS enabled sensor nodes) will be zero or near zero. The sensor nodes should be deployed where the co-ordinates of every point is of relatively opposite sign with respect to the co-ordinate system f_x^A, f_y^A and f_x^B, f_y^B. In Fig. 4, we assume that, $\angle p_n^A p_6 p_n^B = \angle p_n^A p_5 p_n^B = 90^0$. When $\Delta\theta \to 0$, we can assume that the deployment area of WSN nodes with respect to *A* and *B* sets of nodes will be within the rectangle $p_6 p_n^A p_5 p_n^B$. Thus, it can be said that, within $x_n^B < x_k < x_n^A$ and $y_n^B < y_k < y_n^A$, any point $p_k(x_k, y_k)$ can be calculated in error-free manner, or we can control the error significantly. Since $\angle p_n^A p_6 p_n^B = 90^o$ and also $\angle p_n^A p_n^B p_3 = \theta$, we can say $\angle p_n^A p_n^B p_5 = 90^o - \theta$. Let us assume that, the distance between p_n^A and p_n^B is d_n^{AB}. If the distance d_n^{AB} is constant, then the area of rectangle $p_3 p_n^A p_5 p_n^B$ will be $(d_n^{AB})^2 \sin\theta\cos\theta$. Let us assume that, the area of rectangle $p_3 p_n^A p_5 p_n^B$ is denoted by Ar where $Ar = (d_n^{AB})^2 \sin\theta\cos\theta$.

$$Ar = \left(\left(d_n^{AB}\right)^2 \sin 2\theta\right)/2$$

After taking both side derivatives with respect to θ we get:

$$\frac{d}{d\theta}\{Ar\} = (d_n^{AB})^2 \cos 2\theta$$

In the proposed algorithm, our intention is to maximize the number of nodes to become beacon nodes. This is possible only if error in positioning the nodes can be minimized. Since it is assumed that the node density is uniform over the network, therefore, we can say that, if we can maximize the area, then we can keep the average distance same between two sets of beacon nodes. For the maximum or minimum value of total number of nodes (N), we can write:

$$\frac{d}{d\theta}\{Ar\} = 0$$

$$\text{or, } (d_n^{AB})^2 \cos 2\theta = 0$$

$$\text{or, } \cos 2\theta = 0 \text{ since } d_n^{AB} \neq 0$$

$$\theta = \pi(2n+1)/4 \text{ where } n \in \{1,2,3\ldots\} \tag{38}$$

Since $p_3 p_n^A p_5 p_n^B$ is a rectangle, and $\angle p_n^B p_n^A p_3 = \theta$ for that, $\theta \leq \pi/2$. (39)

From (38) and (39)we can say, $\theta = \pi/4$ (40)

Since $\frac{d}{d\theta}\{Ar\}=0$ at $\theta=\frac{\pi}{4}$, thus it can be stated that, when $\theta=\frac{\pi}{4}$ then, Ar will have maximum or minimum value. When $\theta=\frac{\pi}{4}$, then, the area of rectangle (Ar) has maximum value, if and only if:

$$\frac{d^2}{d\theta^2}\{Ar\}<0 \quad (41)$$

$$\text{Now, } \frac{d^2}{d\theta^2}\{Ar\}=-2(d_n^{AB})^2\sin 2\theta \quad (42)$$

$$\frac{d^2}{d\theta^2}\{Ar\}=-2(d_n^{AB})^2 \text{ since } \sin 2\theta=\sin\frac{\pi}{2}=1 \quad (43)$$

Moreover from (43), we can say $\frac{d^2}{d\theta^2}\{Ar\}<0$ (44)

From the above discussion, when $\theta=\frac{\pi}{4}$ then $\frac{d}{d\theta}\{Ar\}=0$ and $\frac{d^2}{d\theta^2}\{Ar\}<0$, then we can say that, at $\theta=\frac{\pi}{4}$, the area of rectangle $p_3p_n^Ap_5p_n^B$ or Ar has the maximum value. Also, when $\theta=\pi/4$ and $p_3p_n^Ap_5p_n^B$ is a rectangle, that indicates $p_3p_n^Ap_5p_n^B$ must be a square. Similar result can be obtained if we place *A* and *B* sets of GPS enabled sensor nodes to the opposite diagonal of square $p_3p_n^Ap_5p_n^B$. The node density per unit area and total number of nodes is denoted by ρ and N respectively. The expression for N is:

$$N=(Ar)\rho \quad (45)$$

If the value of ρ remains constant and since Ar has the maximum value at $\theta=\pi/4$, then N also has the maximum value at $\theta=\pi/4$.

From the above discussion, it can be stated that, by keeping node density same, maximum number of sensor nodes can be localized (with minimum error) if and only if the shape of the network area is a square where AB (distance between the centroid of *A* and *B* sets of GPS-enabled nodes) is a diagonal of the said square network area. Also, the position of *A* and *B* sets of GPS enabled nodes will be such, that AB lines will make $\pi(1+2n)/4$ radians, which means *A* and *B* sets of GPS enabled nodes need to be deployed SW to NE or NW to SE directions.

VIII. Results

We simulated the M-MEMHS algorithm to compare it with MEMHS algorithm. Though the results of theoretical analysis provided better accuracy than simulated results, the trend denotes that the proposed M-MEMHS is the best because it produces most accurate results among the three compared. The proposed M-MEMHS algorithm is simulated using Matlab and the results are presented in the remaining part of this section.

The simulation parameters (Table III, Table IV) are same as these for MEMHS algorithm [7].

TABLE III
NETWORK PARAMETERS

Parameter	Value
Total number of nodes in the network	100
Number of GPS-enabled nodes	6 (Set A: 3 and Set B: 3)
Total area of the network	180 x 180 Sq Meter
Transmission range of sensor node	80 meters

We have simulated the M-MEMHS algorithm based on the radio model described in Table IV [18].

TABLE IV
ENERGY MODEL FOR SIMULATION OF M-MEMHS ALGORITHM

Operation	Energy dissipation
Transmitter Electronics (E_{Tx_elec})	50 nJ/bit
Receiver Electronics (E_{Rx_elec})	
$E_{Tx\_elec}=E_{Rx\_elec}=E_{elec}$	
Transmit Amplifier	1000 pJ/bit/m^2

Fig. 5 represents changes in standard deviation of error with increasing error factor in case of Multilateration, MEMHS and M-MEMHS algorithms. The nature of the graphs for all three cases shows that modular mean of error is least in case of M-MEMHS algorithm with respect to different values of error factor. From equation (23) it can be said that, when the value of error factor equals 1, there is no error in predicting path loss coefficient and the permittivity constant. This is also reflected in Fig. 5, where the modular mean of error is minimum while the value of error factor is equal to 1 for all the algorithms.

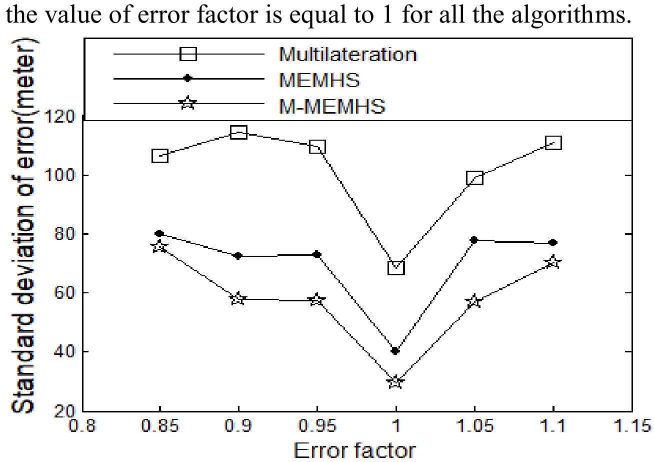

Fig. 5. Standard deviation of error with respect to error factor

Similarly, Fig. 6 shows the variation of average error with varying error factor in case of Multilateration, MEMHS and M-MEMHS algorithms. As evident, in case of average error, the M-MEMHS algorithm shows the best performance.

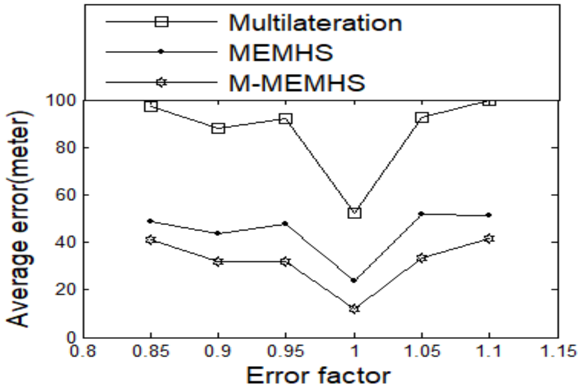

Fig. 6. Average error with respect to error factor

Similarly, Fig. 7 shows the variation of average error with respect to hop distance in case of Multilateration, MEMHS and M-MEMHS algorithms. With average hop distance between 4 and 5, the performance of MEMHS algorithm is better than M-MEMHS algorithm and the reasons behind this can be well explained too. In the M-MEMHS algorithm we did more approximation than MEMHS algorithm. With the approximation, we get advantage by prioritizing the location, which is more accurate than others with the same time approximation. At the middle of the network area when the hop count from both sets of GPS enabled nodes are same, then both locations (with respect to *A* and *B* sets of nodes) get similar priority. Therefore, the hop count parameters cannot be prioritized properly to any result over other. Not only that, but error due to approximation is also more in case of M-MEMHS algorithm, which cannot be cancelled out because of the equal hop count.

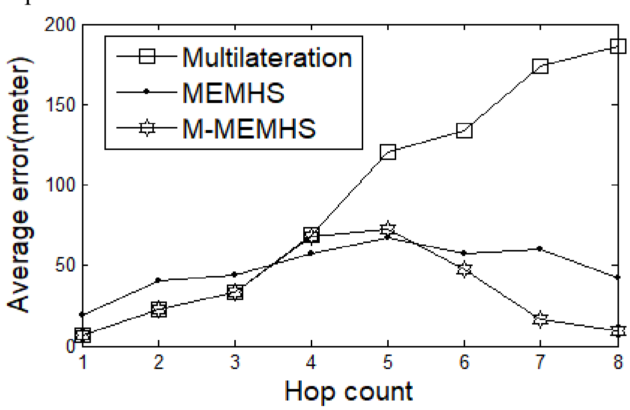

Fig. 7. Average error with respect to hop count

IX. Discussion

Table V shows the comparison of existing localization techniques with respect to proposed M-MEMHS algorithm. The work in [19][20][21] describes centralized algorithms. Generally centralized algorithms do not work very efficiently in hostile environment. Though accuracy may be high, scalability is low and cost is also higher. Schemes proposed in [22][23][24] have lower accuracy, cost and scalability. With less accuracy and scalability, these solutions are not acceptable, though the cost is low. Whereas the schemes in [25][26] are distributed schemes with higher accuracy, but less scalability. The solutions in [27][28] provide higher accuracy and lower cost. However, these schemes are not very scalable and also generate cumulative errors. We proposed a solution for eliminating cumulative error in distributed environment, which supports scalability. Moreover, we showed in the earlier section that the cumulative error is nonlinear in M-MEMHS algorithm.

TABLE V
COMPARISON OF EXISTING PROPOSALS WITH M-MEMHS ALGORITHM

Proposal	Centralized/ Distributed	Accuracy	Cost (Message and Computation)	Scalability
Ref[19]-[21]	Centralized	High	High	Low
Ref [21]-[23]	Distributed	Low	Low	Low
Ref [24]-[25]	Distributed	High	Low	Low
Ref [26][27]	Distributed	High	High	Low
M-MEMHS	Distributed	Moderate	Low	High

X. Conclusion

This paper presents an efficient 2D localization algorithm on top of MEMHS algorithm. In the MEMHS algorithm, the authors proved that, the sign of error is different on either side of neutral point. In case of MEMHS algorithm, authors assumed that, error propagates in linear cumulative way. But error propagates in a nonlinear and cumulative way with respect to the hop count. We have considered this fact and applied modification accordingly and arrived at the M-MEMHS algorithm. In the MEMHS algorithm authors assumed that the magnitude of error does not depend on the angular position of node. But current work proves that magnitude of error is also dependent on the angular position of node with respect to the centroid of GPS enabled nodes (*A* or *B* set). We have modified the MEMHS algorithm by incorporating the previously discussed fact that, error varies with respect to angle of deployment. Equation (32) describes the M-MEMHS algorithm. The present work shows that M-MEMHS algorithm is more efficient than MEMHS algorithm. The simulation results also support this. It is already discussed that the errors are non-linearly related to hop count. In simulation or real-life, we can get more accurate location information, if we can reduce the error occurring from division by small magnitude number. From equations (13) and (17), we can say, s_x^k and s_y^k are the denominators of the function for finding out X and Y coordinates, respectively. It is observed that, for a smaller change in error in the converted beacon node, the percentage of change in s_x^k will be much more in s_y^k (equations (13) and (17)) and vice versa.

This paper discusses optimum deployment strategy of sensor nodes for localizing maximum number of nodes in a most efficient way. In the future, we shall work on the limitation of the algorithm in order to improve efficiency by finding the exact exponential function for getting optimum result. We also look into node deployment strategies for better location accuracy and increased scalability.

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